Warm-up!

The function f is given by $f(x) = \sin(2x) - \sin(x)$.

A) Rewrite f using the "sine double angle formula" we learned before break.

$f(x) = 2\sin(x)\cos(x) - \sin(x)$

B) Starting from the rewritten version of this function, solve $f(x) = 0$ over the interval $[0, 2\pi)$.

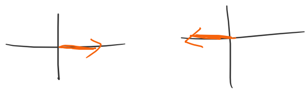
$2\sin(x)\cos(x) - \sin(x) = 0$

$(\sin x)(2\cos x - 1) = 0$

$\sin x = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$



$\theta = 0$ $\quad \theta = \pi$ $\quad \theta = \frac{\pi}{3}$ $\quad \theta = \frac{5\pi}{3}$ $\quad 2\pi - \frac{\pi}{3}$

C) Solve $f(x) = 0$ over the interval $[-\pi, \pi]$

$\theta = 0$ $\quad \theta = -\pi$ $\quad \theta = \frac{\pi}{3}$ $\quad \theta = -\frac{\pi}{3}$

D) Find all input values in the domain of f for which $f(x) = 0$.

$\theta = 0 + 2\pi k$ $\quad \theta = \pi + 2\pi k$ $\quad \theta = \frac{\pi}{3} + 2\pi k$ $\quad \theta = \frac{5\pi}{3} + 2\pi k$, where k is any integer

$\theta = \pi k$

What we're doing today

- Remembering how to solve trig equations

- Remembering what it means to find all solutions. This includes how sometimes we write $+\pi n$ rather than $+2\pi n$.

- Practicing the formulas we learned last class

- Review solving inverse trig equations

The Two/Six/Eight Formulas We Learned Before Break. With each problem we do today involving these, try to remember the formula on your own before looking back at note to verify you got it right.

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$
$$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$$

$$\sin(2\theta)=2\sin\theta\cos\theta$$

$$\cos(2\theta)=\begin{cases}\cos^2\theta-\sin^2\theta\\ 2\cos^2\theta-1\\ 1-2\sin^2\theta\end{cases}$$

The function g is given by $g(x) = \dfrac{\sin\left(\frac{\pi}{2} - x\right)}{\sin(\pi - x)}$ $*$

A) Rewrite $g(x)$ as a single trigonometric function.

$$g(x) = \frac{\sin\left(\frac{\pi}{2}\right)\cos(x) - \cos\left(\frac{\pi}{2}\right)\sin(x)}{\sin(\pi)\cos(x) - \cos(\pi)\sin(x)}$$

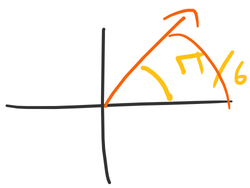
$$g(x) = \frac{1 \cdot \cos x - 0 \cdot \sin x}{0 \cdot \cos x - (-1)\sin x} = \frac{\cos x}{\sin x} = \cot x$$

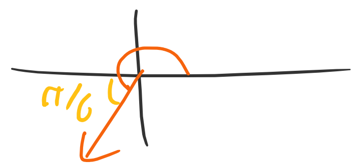
B) Find all input values in the domain of g that yield an output value of $\sqrt{3}$.

$$\cot x = \sqrt{3}$$

$\cot x = \sqrt{3}$

$\tan x = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$





$x = \dfrac{\pi}{6} + 2\pi k$ $\qquad$ $x = \dfrac{7\pi}{6} + 2\pi k, \; k \in \mathbb{Z}$

$$x = \frac{\pi}{6} + \pi k, \; k \in \mathbb{Z}$$

Let $f(x) = 5\sin x - \cos(2x)$

$$\cos(2\theta) = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

A) Rewrite f as a function of sine only

$f(x) = 5\sin x - (1 - 2\sin^2 x)$

$f(x) = 2\sin^2 x + 5\sin x - 1$

B) Find all values in $[0,2\pi)$ that solve $f(x) = 2$

$2\sin^2 x + 5\sin x - 1 = 2$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -3$ impossible

$x = \frac{\pi}{6}$ $\qquad$ $x = \frac{5\pi}{6}$

$2A^2 + 5A - 1 = 2$

$2A^2 + 5A - 3 = 0$

$(2A - 1)(A + 3) = 0$

$A = \frac{1}{2}$ $\qquad$ $A = -3$

C) Find all values that solve $f(x) = 2$

$x = \frac{\pi}{6} + 2\pi k$ $\qquad$ $x = \frac{5\pi}{6} + 2\pi k$, where k is any integer

A reminder about inverse trigonometric functions

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\arcsin(x)$

Sometimes written $\sin^{-1} x$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Inverse function for $\sin(x)$

$-\frac{\pi}{2}$ $\frac{\pi}{2}$

$y = \sin x$ domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin x$ range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arcsin(-1) = -\frac{\pi}{2}$

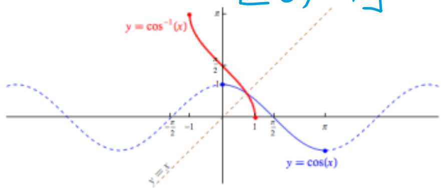
$\arccos(x)$

Sometimes written $\cos^{-1} x$

Range: $[0, \pi]$

Inverse function for $\cos(x)$

$[0, \pi]$



$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\arctan(x)$

Sometimes written $\tan^{-1} x$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Inverse function for $\tan(x)$

The function f is given by $f(\theta) = \cos(2\theta) + \cos(\theta) - \sin^2(\theta)$

$f(\theta) = (2\cos^2\theta - 1) + \cos\theta - (1 - \cos^2\theta)$

Rewrite f using only $\cos\theta$

$f(\theta) = 3\cos^2\theta + \cos\theta - 2$

Of the following, which are zeros of f?

(A) 0

(B) $\frac{\pi}{2}$

(C) π

(D) $\frac{3\pi}{2}$

(E) $\arccos\left(\frac{2}{3}\right)$

(F) $\pi - \arccos\left(\frac{2}{3}\right)$

(G) $\pi + \arccos\left(\frac{2}{3}\right)$

(H) $2\pi - \arccos\left(\frac{2}{3}\right)$

(I) $2\pi + \arccos\left(\frac{2}{3}\right)$

$3\cos^2\theta + \cos\theta - 2 = 0$

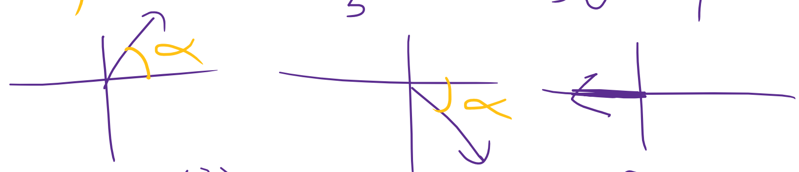
$3A^2 + A - 2 = 0$

$(3A - 2)(A + 1) = 0$

$A = \frac{2}{3}$ $\quad A = -1$

$\alpha = \arccos\left(\frac{2}{3}\right)$

$\cos\theta = \frac{2}{3}$ $\quad \cos\theta = -1$



$\arccos\left(\frac{2}{3}\right)$ $\quad 2\pi - \arccos\left(\frac{2}{3}\right)$ $\quad \theta = \pi + 2\pi k$

Inverse Trig Equations

$$2\boxed{\arccos(x-1)} = \frac{5\pi}{3}$$

$$\arccos(x-1) = \frac{5\pi}{6}$$

$$\cos\left(\arccos(x-1)\right) = \cos\left(\frac{5\pi}{6}\right)$$

$$x-1 = -\frac{\sqrt{3}}{2}$$

$$x = -\frac{\sqrt{3}}{2} + 1$$

$$4\sin^{-1}\left(x\sqrt{2}\right) = -\pi$$

$$4\boxed{\arcsin(x\sqrt{2})} = -\pi$$

$$\sin\left(\arcsin(x\sqrt{2})\right) = \sin\left(-\frac{\pi}{4}\right)$$

$$x\sqrt{2} = -\frac{1}{\sqrt{2}}$$

$$x = -\frac{1}{2}$$

$$\arctan\left(\frac{x}{4}\right) = \frac{3\pi}{4}$$

impossible

$$\cancel{\tan\left(\arctan\left(\frac{x}{4}\right)\right) = \tan\left(\frac{3\pi}{4}\right)}$$

$$\cancel{\frac{x}{4} = -1}$$

$$\cancel{x = -4}$$

Inverse Trig Equations

The function f is given by $f(x) = 10\arcsin(2x)$.
Solve $f(x) = 5\pi$ for values of x in the domain of f.

$$5\pi = 10\boxed{\arcsin(2x)}$$
$$\frac{\pi}{2} = \boxed{\arcsin(2x)}$$
$$\sin\left(\frac{\pi}{2}\right) = \sin(\arcsin(2x))$$
$$1 = 2x$$
$$\frac{1}{2} = x$$

The function h is given by $h(x) = 3\pi - 6\arctan\left(\frac{x}{2}\right)$.
Evaluate $h^{-1}(5\pi)$.

$$\to \boxed{h} \to 5\pi$$
$$5\pi \to \boxed{h^{-1}} \to$$

$$5\pi = 3\pi - 6\arctan\left(\frac{x}{2}\right)$$
$$2\pi = -6\arctan\left(\frac{x}{2}\right)$$
$$\tan\left(-\frac{\pi}{3}\right) = \tan\left(\arctan\left(\frac{x}{2}\right)\right)$$
$$-\sqrt{3} = \frac{x}{2}$$
$$-2\sqrt{3} = x$$

Find some solutions vs find all solutions

The function g is given by $g(x) = \sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right)$.

Find solutions to $g(x) = 0$ in the interval $[0, \pi]$.

Find solutions to $g(x) = 0$ in the interval $[-\pi, 3\pi]$.

Find all solutions to $g(x) = 0$.

Inverse Trig Equations

The function r is given by $r(x) = \frac{\tan^{-1}(x+4)}{2}$.
Solve $r(x) = \arcsin\left(\frac{1}{2}\right)$.

The function t is given by $t(x) = \cos^{-1}(-10x)$. Find all input values in the domain of t that yield an output value of $\arctan(1)$.

Find all solutions

The function g is given by $g(\theta) = 6\sin^2\theta + 4\sin\theta + \cos(2\theta)$.
Find all values in the domain of g that yield an output value of 0.

$$6\sin^2\theta + 4\sin\theta + (1 - 2\sin^2\theta)$$

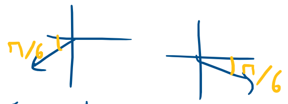
$$0 = 4\sin^2\theta + 4\sin\theta + 1$$

$$0 = 4A^2 + 4A + 1$$

$$0 = (2A+1)(2A+1)$$

$$A = -\tfrac{1}{2} \qquad A = -\tfrac{1}{2}$$

$$\sin\theta = -\tfrac{1}{2}$$



$$\theta = \frac{7\pi}{6} + 2\pi k \qquad \theta = \frac{11\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}.$$

Write an expression for all solutions to $\dfrac{\cos\left(\theta - \frac{\pi}{2}\right)}{1 - \cos^2\theta} = 2$

$$\frac{\cos(\theta)\cos\left(\frac{\pi}{2}\right) + \sin(\theta)\sin\left(\frac{\pi}{2}\right)}{1 - \cos^2\theta} = 2$$

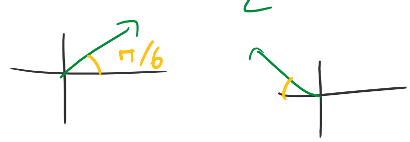
$$\frac{\cos\theta \cdot 0 + \sin\theta \cdot 1}{1 - \cos^2\theta} = 2$$

$$\frac{\sin\theta}{1 - \cos^2\theta} = 2$$

$$\frac{\sin\theta}{\sin^2\theta} = 2$$

$$\frac{1}{\sin\theta} = 2$$

$$\sin\theta = \frac{1}{2}$$



$$\theta = \frac{\pi}{6} + 2\pi k \qquad \theta = \frac{5\pi}{6} + 2\pi k$$

, where k is any integer